

Mechanism Design for Collective Choice

Master 2

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Level : Master second year M2 / first semester

Cursus: Economics, Data and Decision Science: 21h in class (lectures)

Teacher: Philippe Solal

Overview

This course is divided into three parts. The first part concerns the theory of social choice and presents some impossibility results and an axiomatic characterization of the majority rule. The second part studies the possibility of implementing social choice functions in Nash equilibrium. The third part applies the results obtained above to the one-to-one matching problem. Of particular interest are social choice functions that select stable matchings.

Skills developed

Social choice theory defines the social goals a community wants to achieve. Social goals are defined on the basis of principles of efficiency, justice, consistency, etc., and are embodied in a social choice function, i.e. a centralized mechanism. Implementation theory studies the possibilities of decentralizing the social choice function (the social goals) through an incentive mechanism in which rational agents can exploit strategic opportunities. So the question is, under what conditions can the social choice function be decentralized by such an incentive mechanism? And if such a mechanism exists, what form can it take (a bargaining procedure, an auction procedure, an exchange procedure . . .)? In particular contexts such as the one-to-one matching problem, is it possible to implement stable social choice functions?

Content

1 Introduction

- 1.1 Social choice theory and its axiomatic method
- 1.2 Social states, preferences, social goals and mechanisms
- 1.3 The Mount-Reiter diagram
- 1.4 An example: King Solomon's dilemma
- 1.5 Matching theory and Social choice theory

2 Social choice theory

- 2.1 The basic structure of the social choice theory
- 2.2 Axioms for social choice functions
- 2.3 An axiomatic characterization of the Majority rule
- 2.4 Some impossibility results
 - 2.4.1 The impossibility of a Paretian liberal

- 2.4.2 The Muller-Satterthwaite theorem
- 2.4.3 The Arrow impossibility theorem

3 Mechanism design

- 3.1 The basic structure of the mechanism design problem
- 3.2 Axiomatic foundation of Nash equilibrium
 - 3.2.1 Solution concept for strategic games
 - 3.2.1 Axiomatic approach to the Nash equilibrium solution
- 3.3 Nash implementation
 - 3.3.1 Incentive compatibility and implementation
 - 3.3.2 Maskin's result.

4 One-to-one matching problems

- 4.1 Matching problem as a social choice problem
- 4.2 Stable matchings
- 4.3 The lattice structure of stable matchings
- 4.4 Uniqueness of the stable matching
- 4.5 The Core of a one-to-one matching problem and its set of stable matchings

5 Nash implementation

- 5.1 Nash implementation of the stable social choice function
- 5.2 An impossibility result for resolute and stable social choice functions
- 5.3 The Gale-Shapley algorithm
 - 5.3.1 The Deferred Acceptance algorithm
 - 5.3.2 Optimal stable matchings
 - 5.3.3 Strategy-proofness

6 References

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Prerequisites

Economics of Bargaining and collective choice course at Master 1 level.

Grading

Written exam (2 hours)